I Semester [Digital Electronics]

USN | | | | | | | | | | |

12EC046

M.Tech. Degree Examination, Dec.2013/Jan.2014

Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Define field. Find the general solution of the linear system whose augmented matrix is

$$A=\begin{bmatrix}1 & 6 & 2 & -5 & -2 & -4\\ 0 & 0 & 2 & -8 & -1 & 3\\ 0 & 0 & 0 & 0 & 1 & 7\end{bmatrix}.$$ **(10 Marks)**

b. Solve the equation AX = b by using LU factorization, given

$$A=\begin{bmatrix}2 & -1 & 2\\ -6 & 0 & -2\\ 8 & -1 & 5\end{bmatrix},\ b=\begin{bmatrix}1\\ 0\\ 4\end{bmatrix}.$$ **(10 Marks)**

2 a. Define basis and dimension of a vector space. Show that the set B = {(1.1.0), (1.0.1), (0.1.1.)} is a basis for the vector space $V_3(R)$. **(07 Marks)**

b. If W_1 and W_2 are finite dimensional subspace of a vector space V, $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$. **(07 Marks)**

c. Prove that the non zero rows of matrix in echelon form are linearly independent. **(06 Marks)**

3 a. Let V and W be vector spaces over the field F and T be a linear transformation from V into W. Suppose that V is finite-dimensional. Then prove that rank (T) + nullity (T) = dimV. **(10 Marks)**

b. Let V and W be vector space over the field F and T be a linear transformation from V into W. If T is invertible then the inverse function T^{-1} is a linear transformation from W onto V. **(10 Marks)**

4 a. Prove that every vector space V over the real field R of dimension n is isomorphic $V_n(R)$. **(08 Marks)**

b. Explain:

i) Annihilating polynomial.
ii) Direct sum decomposition.
iii) Minimal polynomial.
iv) Principal decomposition. **(12 Marks)**

5 a. State and prove primary decomposition theorem. **(10 Marks)**

b. Explain Jordan canonical form. **(10 Marks)**

6 a. Explain Gram-Schmidt process. Apply Gram-Schmidt process to find the orthogonal basis and orthonormal basis for the subspace of u spanned by X_1 = [1.1.1.1], X_2 = [0.1.1.1], X_3 = [0.0.1.1]. **(10 Marks)**

b. Find the QR factorization of $A = \begin{bmatrix} 1 & 5 & -9 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$. **(10 Marks)**

7 a. Find the least square solution of AX = B for

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}.$$ **(10 Marks)**

b. Orthogonally diagonalize the matrix

$$A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}.$$ **(10 Marks)**

8 a. Make a change of variable that transforms the quadratic form of $Q(x) = x_1^2 - 8x_1x_2 - 5x_2^2$ into a quadratic form with no cross product term. **(10 Marks)**

b. Explain singular value decomposition. Find the maximum and minimum value of $Q(x) = 9x_1^2 + 4x_2^2 + 3x_3^2$ subject to the constraints $x^T x = 1$. **(10 Marks)**

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